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HARMONIC ALGORITHM GMDH FOR LARGE DATA VOLUME

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In the following paper the application domain of harmonic algorithm GMDH is discussed.
The inconsistency of MLS-estimation of an amplitude vector, used in harmonic algorithm of GMDH, as a result of the evaluation error in the frequency vector is proved. We have evaluated the convergence rate of the norm of amplitude vector MLS-estimate to zero under the condition that data volume increases without bounds depending on the error evaluation in the frequency vector. We have also estimated the largest data volume, for which using of GMDH harmonic algorithm is still correct.

KEY WORDS Modelling, GMDH-technique, harmonic analysis.

The GMDH algorithms for modelling of polyharmonic processes and fields [1–3] are widely used. These algorithms rest upon the idea of the inductive self-organiza tion principle and choice of the best model in sense of some discrimination criterion. The algorithm for one-dimensional process modelling is aimed at the model structure search (and parameter estimation) in the following class of the model structures:

$$F = \left\{ y_t = a_0 + \sum_{k=1}^{m} (a_k \sin \omega_k t + b_k \cos \omega_k t) \right\}, m \in M$$

where a_k, b_k, ω_k are model parameters. The model structure is defined by the value of m (number of harmonics), M being the set of permissible values of harmonics number m.

The GMDH harmonic algorithm employs the following scheme: at first, for each fixed $m \in M$ the model parameters are estimated and then the obtained models are compared by means of the discrimination criterion. To use such a scheme one must solve two problems:

a) the construction of an effective algorithm for parameter estimation;
b) the choice of a discrimination criterion (the criteria of model quality).

In this paper we shall discuss the problem of parameter estimation. In GMDH harmonic algorithm the estimation of parameters is carried out under the assumption that the model has an unbiased structure (value m is true). Suppose that the process model is the sum of m harmonic components with pairwise distinct frequencies $\omega_1, \omega_2, \ldots, \omega_m$

$$y_t = a_0 + \sum_{k=1}^{m} (a_k \sin \omega_k t + b_k \cos \omega_k t) \tag{1}$$

where a_k, b_k are coefficients and $\omega_i \neq \omega_j$, $i \neq j$, $0 < \omega_k < \pi$, $1 \leqslant k \leqslant m$. The process is observed at equal discrete time intervals at the points t, $1 \leqslant t \leqslant N$. The observed sequence is z_t:

$$z_t = y_t + \delta_t, \quad 1 \leqslant t \leqslant N$$

where δ_t is the sequence of similarly distributed independent random variables with zero mean and unknown finite variance ($\sigma^z < \infty$).

The well known approach to parameter estimation based on minimizing the norm $\| Y - Z \|$, where $Y = (y_1, \ldots, y_N)^T$, $Z = (z_1, \ldots, z_N)^T$ is not suitable for harmonic models as the parameters ω_k are contained nonlinearly in function (1) and there is no finite stable procedure for solving the above mentioned problem. Therefore, in the GMDH harmonic algorithm, parameters are estimated by the three stage scheme [1,3]. Here we will describe this scheme briefly. Note that this scheme is intended for parameter estimation in models without the constant term a_0,

$$y'_t = \sum_{k=1}^{m} (a'_k \sin \omega_k t + b'_k \cos \omega_k t), \quad 1 \leqslant t \leqslant N. \tag{2}$$

One can readily see that the variables $y'_t = y_{t+1} - y_{t-1}$ satisfy equalities (2) in which

$$a'_k = -2b_k \cos \omega_k t, \quad b'_k = 2a_k \sin \omega_k t.$$

At the first stage of the described scheme the coefficients of difference equation

$$y'_t = \sum_{k=1}^{m} \alpha_k (y_{t+k} - y_{t-k}) \tag{3}$$

are estimated. The process (2) always satisfies the equality (3). The inverse statement is not always true. Equality (3) has the solution (2) if and only if the roots of the secular equation, corresponding to difference equation (3), are pairwise distinct and not greater than 1. One must note the fact that if the variables y'_t satisfy equalities (2) then there exists the unique collection $\alpha_1, \alpha_2, \ldots, \alpha_m$ for which equality (3) is true for every integer k.

At the second stage the frequencies $\omega_1, \omega_2, \ldots, \omega_m$ are evaluated from the equation

$$2 \sum_{k=1}^{m} \hat{\alpha}_k \cos k\omega - 1 = 0, \tag{4}$$

where as $\hat{\alpha}_k$ designates the estimations of coefficients from the first stage. The equality (4) always has not more than m roots and if the number of solutions is exactly m then those vlaues can be used as the estimations $\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_m$ of true frequencies.

Finally at the third stage, the parameters a_k, b_k are estimated. The background of the described three stages procedure can be found in [3].

In [3] the author shows that the observations

$$z_t' = z_{t+1} - z_{t-1}$$

cannot be used directly in the described scheme for the reason that though the random variables $\delta_t' = z_t - y_t$ have the zero mean and equal variances $2\sigma^2$, in contrast to the random variables δ_t they are not independent, $\operatorname{cov}(\delta_{t+1}', \delta_{t-1}') = -\sigma^2$. In this work the author proves the inconsistency of MLS-estimation for the coefficients $\alpha_1, \alpha_2, \ldots, \alpha_m$ in equality (3) when variables z_t' are used at the first stage. Moreover he proves that the orthogonal regression gives the consistent estimation of the coefficients $\alpha_1, \alpha_2, \ldots, \alpha_m$.

As frequency estimates are computed directly from relation (4), the consistency of the estimates $\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_m$ follows by the consistency of the estimates of coefficients of difference equation (3) $\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_m$. In the existing algorithms at the third stage, the amplitudes $a_k, b_k (1 \leqslant k \leqslant m)$ are estimated by the least square method.

The described GMDH harmonic algorithm was used successfully for the solution of a variety of modelling problems. However, our efforts to employ the algorithm for the solution of modelling problems in case of large samples proved not to be effective. It turned out that even with good frequency estimates the amplitude estimates were close to zero regardless of their true values. This failures motivated us to investigate the amplitude performance for large data samples when there are errors in frequency vector evaluation. The results of the investigation may be presented by means of the following assertion.

Suppose that the values

$$y_t = a_0 + \sum_{k=1}^{m} (a_k \sin \omega_k t + b_k \cos \omega_k t) + \delta_t, \quad 1 \leqslant t \leqslant N$$

are observed. Let also for MLS-estimation of the amplitude vector $(a_1, b_1, \ldots a_m, b_m)^T$, an estimate of the frequency vector $\omega = (\omega_1 + \varepsilon_1, \omega_2 + \varepsilon_2, \ldots, \omega_m + \varepsilon_m)^T$ be used, where $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m)^T$ is the estimation error vector.

Assertion If the vector $\varepsilon \not\equiv 0$, then the MLS-estimate of the amplitude vector is inconsistent. If $\varepsilon_k \neq 0$, then MLS-estimates of the corresponding components of the amplitude vector converge to zero and

$$\begin{bmatrix} \hat{a}_k \\ \hat{b}_k \end{bmatrix} = 0\left(\frac{1}{N^{1/2-p}}\right) \quad \text{for any } p \geqslant 0.$$

Proof: Denote by X the $n \times 2m$-dimension matrix

$$X = \begin{bmatrix} \sin \hat{\omega}_1 & \cos \hat{\omega}_1 & \ldots & \sin \hat{\omega}_m & \cos \hat{\omega}_m \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \sin \hat{\omega}_1 t & \cos \hat{\omega}_1 t & \cdots & \sin \hat{\omega}_m t & \cos \hat{\omega}_m t \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \sin \hat{\omega}_1 N & \cos \hat{\omega}_1 N & \ldots & \sin \hat{\omega}_m N & \cos \hat{\omega}_m N \end{bmatrix}.$$

Let Y be the vector of observations

$$Y=\begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ Y_t \\ \cdot \\ \cdot \\ Y_N \end{bmatrix}=\begin{bmatrix} \sum_{k=1}^{m}(a_k \sin\omega_k + b_k\cos\omega_k)+\delta_1 \\ \cdots \\ \sum_{k=1}^{m}(a_k\sin\omega_k t+b_k\cos\omega_k t)+\delta_t \\ \cdots \\ \sum_{k=1}^{m}(a_k\sin\omega_k N+b_k\cos\omega_k N)+\delta_N \end{bmatrix}$$

To obtain the MLS-estimation of amplitude vector one can use the well known relation:

$$(\hat{a}_1,\hat{b}_1,\cdots \hat{a}_m,\hat{b}_m)^T=(X^TX)^{-1}X^TY.$$

First, let us prove the statement in case $m=1$, and then show how this case can be extended to a general case of arbitrary m. To make the proof look simpler in the case $m=1$ we omit the subscripts in expressions $a_1, b_1, \omega_1, \varepsilon_1, \hat{a}_1, \hat{b}_1, \hat{\omega}_1$.

For $m=1$ one can write the (X^TX) matrix in the form

$$X^TX=\begin{bmatrix} \sum_{t=1}^{N}\sin^2\hat{\omega}t & \sum_{t=1}^{m}\sin\hat{\omega}t\cos\hat{\omega}t \\ \sum_{t=1}^{m}\sin\hat{\omega}t\cos\hat{\omega}t & \sum_{t=1}^{m}\cos^2\hat{\omega}t \end{bmatrix}.$$

Using trignometric relations one can calculate the sums in X^TX matrix and write out the inverse matrix

$$(X^TX)^{-1}=\frac{\sin\hat{\omega}}{N^2\sin^2\hat{\omega}-\sin^2(N\hat{\omega})}$$

$$\times\begin{bmatrix} (2N-1)\sin\hat{\omega}+\sin(2N+1)\hat{\omega} & \cos(2N+1)\hat{\omega}-\cos\hat{\omega} \\ \cos(2N+1)\hat{\omega}-\cos\hat{\omega} & (2N+1)\sin\hat{\omega}-\sin(2N+1)\hat{\omega} \end{bmatrix}.$$

We are interested in MLS-estimation when N increases without bounds, therefore we rewrite the $(X^TX)^{-1}$ matrix in the form

$$(X^TX)^{-1}=\frac{1}{N}\begin{bmatrix} 2+0\left(\frac{1}{N}\right) & 0\left(\frac{1}{N}\right) \\ 0\left(\frac{1}{N}\right) & 2+0\left(\frac{1}{N}\right) \end{bmatrix}.$$

The X^TY vector may be presented as the sum

$$X^TY=\begin{bmatrix}\sum_{t=1}^{m}(a\sin\omega t\sin\hat{\omega}t+b\cos\omega t\sin\hat{\omega}t)\\ \sum_{t=1}^{m}(a\sin\omega t\cos\hat{\omega}t+b\cos\omega t\cos\hat{\omega}t)\end{bmatrix}+\begin{bmatrix}\sum_{t=1}^{m}\delta_t\sin\hat{\omega}t\\ \sum_{t=1}^{m}\delta_t\cos\hat{\omega}t\end{bmatrix}$$

When the sums in the first summand are calculated we get the X^TY vector

$$X^TY=\frac{a}{4}\begin{bmatrix}\dfrac{\sin(2N+1)\varepsilon/2}{\sin\varepsilon/2}-\dfrac{\sin(2N+1)(\omega+\varepsilon/2)}{\sin(\omega+\varepsilon/2)}\\ \dfrac{\cos(\omega+\varepsilon/2)-\cos(2N+1)(\omega+\varepsilon/2)}{\sin(\omega+\varepsilon/2)}-\dfrac{\cos\varepsilon/2-\cos(2N+1)\varepsilon/2}{\sin\varepsilon/2}\end{bmatrix}$$

$$+\frac{b}{4}\begin{bmatrix}\dfrac{\cos(\omega+\varepsilon/2)-\cos(2N+1)(\omega+\varepsilon/2)}{\sin(\omega+\varepsilon/2)}+\dfrac{\cos\varepsilon/2-\cos(2N+1)\varepsilon/2}{\sin\varepsilon/2}\\ \dfrac{\sin(2N+1)\varepsilon/2}{\sin\varepsilon/2}-\dfrac{\sin(2N+1)(\omega+\varepsilon/2)}{\sin(\omega+\varepsilon/2)}\end{bmatrix}$$

$$+\begin{bmatrix}\sum_{t=1}^{m}\delta_t\sin\hat{\omega}t\\ \sum_{t=1}^{m}\delta_t\cos\hat{\omega}t\end{bmatrix}.$$

Note that such trignometric relations can be used only when $\varepsilon\neq 0$ and $(\omega+\varepsilon/2)\neq 0$. But these restrictions are obviously fulfilled, as we have supposed that $\varepsilon\neq 0$, and the error in frequency estimation is essentially less than the frequency value ($\varepsilon\ll\omega$).

Let's introduce the next designations

$$A_1^N=\frac{\sin(2N+1)\varepsilon/2}{\sin\varepsilon/2}$$

$$A_2^N=\frac{\sin(2N+1)(\omega+\varepsilon/2)}{\sin(\omega+\varepsilon/2)}$$

$$B_1^N=\frac{\cos\varepsilon/2-\cos(2N+1)\varepsilon/2}{\sin\varepsilon/2}$$

$$B_2^N=\frac{\cos(\omega+\varepsilon/2)-\cos(2N+1)(\omega+\varepsilon/2)}{\sin(\omega+\varepsilon/2)}.$$

It can easily be seen that the functions A_1^N, A_2^N, B_1^N and B_2^N are bounded by the constant $2/(\sin\varepsilon/2)$, which is independent of N.

Using these designations one can rewrite vector $X^T Y$ as follows:

$$X^T Y = \frac{a}{4}\begin{bmatrix} A_1^N - A_2^N \\ B_2^N - B_1^N \end{bmatrix} + \frac{b}{4}\begin{bmatrix} B_2^N - B_1^N \\ A_1^N - A_2^N \end{bmatrix} + \begin{bmatrix} \sum_{t=1}^{N} \delta_t \sin \hat{\omega} t \\ \sum_{t=1}^{N} \delta_t \cos \hat{\omega} t \end{bmatrix},$$

the MLS-estimation of the amplitude vector can be written in the form

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \frac{1}{2N}\begin{bmatrix} a(A_1^N - A_2^N) + b(B_2^N - B_1^N) + 0\left(\frac{1}{N}\right) \\ \cdot \\ a(B_2^N - B_1^N) + b(A_1^N - A_2^N) + 0\left(\frac{1}{N}\right) \end{bmatrix} + \Delta^N,$$

where

$$\Delta^N = \begin{bmatrix} 2 + 0\left(\frac{1}{N}\right) & 0\left(\frac{1}{N}\right) \\ 0\left(\frac{1}{N}\right) & 2 + 0\left(\frac{1}{N}\right) \end{bmatrix} \begin{bmatrix} \dfrac{\sum_{t=1}^{N} \delta_t \sin \hat{\omega} t}{N} \\ \dfrac{\sum_{t=1}^{N} \delta_t \cos \hat{\omega} t}{N} \end{bmatrix}.$$

The summand Δ^N defines how much MLS-estimation puts down the noise in harmonic algorithm when vector $\hat{\omega}$ is fixed.

Now we will show that the MLS-estimation of the amplitude vector tends to zero and hence is not consistent. Firstly, we will prove that Δ^N converges to zero and we will estimate the convergence rate. The required results can be derived from the following [4].

If S_N is the sum of N independent random variables with zero means and finite variances, and the variance of the sum DS_N increases without bounds then the mean of the sum S_N converges to zero and convergence rate is estimated as $0((DS_n)^{1/2+p})$ with probability 1, where p is an arbitrary positive value.

It follows from this statement that the means of the sums $\sum_{t=1}^{N} \delta_t \sin \hat{\omega} t$ and $\sum_{t=1}^{N} \delta_t \cos \hat{\omega} t$ converge to zero and the convergence rate can be estimated by a function $0(N^{1/2+p})$ with probability 1.

It is easy to evaluate the variances of the sum

$$\sigma^2 \sum_{t=1}^{N} \delta_t \sin^2 \hat{\omega} t = \sigma^2 (K_1 N + C_1)$$

$$\sigma^2 \sum_{t=1}^{N} \delta_t \cos^2 \hat{\omega} t = \sigma^2 (K_2 N + C_2)$$

where K_1, C_1, K_2, C_2, are constants.

Since one can get the desired estimates

$$\frac{\sum_{t=1}^{N} \delta_t \sin \hat{\omega} t}{N} = 0\left(\frac{1}{N^{1/2-p}}\right) \quad \text{with probability 1}$$

and

$$\frac{\sum_{t=1}^{N} \delta_t \cos \hat{\omega} t}{N} = 0\left(\frac{1}{N^{1/2-p}}\right) \quad \text{with probability 1}$$

where $p \geqslant 0$.

Thus Δ^N is a random vector with zero mean and it is estimated by a function $0(1/N^{1/2-p})$, $p \geqslant 0$ with probability 1.

The MLS-estimation of the amplitudes is also a random variable and now it becomes evident that its mean is defined by the value

$$\frac{1}{2N}\begin{bmatrix} a(A_1^N - A_2^N) + b(B_2^N - B_1^N) + 0\left(\frac{1}{N}\right) \\ a(B_2^N - B_1^N) + b(A_1^N - A_2^N) + 0\left(\frac{1}{N}\right) \end{bmatrix}.$$

It can be readily seen that this summand also converges to zero, when the error ε is fixed and $\varepsilon \neq 0$. As it was mentioned above the functions A_1^N, A_2^N, B_1^N, B_2^N are bounded by $2/(\sin \varepsilon/2)$ and the convergence rate is estimated by a function $0(1/N)$.

Thus, the MLS-estimation of an amplitude vector is not consistent when $\varepsilon \neq 0$. The rate of its convergence to zero is defined by the summand Δ^N and estimated by a function $0(1/N^{1/2-p})$, $p \geqslant 0$ with probability 1. But it must be noted that the requirement $\varepsilon \neq 0$ is essential. If $\varepsilon = 0$ then MLS-estimation is absolutely unbiased

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \Delta^N.$$

Note that the result obtained for $m = 1$ is sufficient to prove the inconsistency of amplitude MLS-estimation in general.

The proof of the assertion for m different from 1 involves some awkward trigonometric transformations and is based on three facts:

1) $2m \times 2m$ – dimensional matrix $(X^T X)^{-1}$ for arbitrary m can be presented as

$$(X^T X)^{-1} = \frac{1}{N}\begin{bmatrix} 2 + 0\left(\frac{1}{N}\right) & & 0\left(\frac{1}{N}\right) \\ & \ddots & \\ 0\left(\frac{1}{N}\right) & & 2 + 0\left(\frac{1}{N}\right) \end{bmatrix}_{2m \times 2m}$$

2) for any $\varepsilon_k \neq 0$, the corresponding elements of $E(X^T Y)$ vector $E(X^T Y)_{2k-1}$ and $E(X^T Y)_{2k}$ are bounded by a constant, which depends on ω and ε and is independent of N.
3) vector Δ^N can be written in the form $\Delta^N = (\Delta_1^N, \Delta_2^N, \ldots, \Delta_m^N)^T$, where

$$(\Delta_k^N)^T = \begin{bmatrix} 2 + 0\left(\frac{1}{N}\right) & 0\left(\frac{1}{N}\right) \\ 0\left(\frac{1}{N}\right) & 2 + 0\left(\frac{1}{N}\right) \end{bmatrix} \begin{bmatrix} \dfrac{\sum_{t=1}^{N} \delta_t \sin \hat{\omega}_k t}{N} \\ \dfrac{\sum_{t=1}^{N} \delta_t \cos \hat{\omega}_k t}{N} \end{bmatrix};$$

therefore $\Delta^N = 0(1/N^{1/2-p})$, $p \geqslant 0$ with probability 1.

The facts 1)–2) are true only under assumption that $\omega_i \neq \omega_j$ $(i \neq j)$ and $\varepsilon \neq 0$.

Thus we show that even if one of the vector ω elements is evaluated with an error (i.e., $\varepsilon \not\equiv 0$) then the MLS-estimation of the amplitude vector is inconsistent and the corresponding elements of the amplitude vector converge to zero when the data volume increases without bounds.

We must note that the GMDH harmonic algorithm was successfully used for solving different modelling problems. To investigate the properties of harmonic algorithm the authors performed a large number of computational experiments but the in- consistency of MLS-estimation of amplitude vector was never detected. All of the authors get sufficiently good results in amplitude estimation when the noise values are small and the model structure (the number of harmonics) is true. But nobody has conducted tests on the large data volume. On the contrary, most of the authors investigate the modelling difficulties when the data sample volume is small.

To research the application domain of GMDH harmonic algorithm we have performed the additional investigations.

First we consider MLS-estimation of an amplitude vector mean in the case $m = 1$ when the error in frequency evaluation is small. One can easily get

$$E(X^T Y) = \frac{1}{4 \sin \varepsilon/2} \begin{bmatrix} a \sin(2N+1)\varepsilon/2 + b(1 - \cos(2N+1)\varepsilon/2) + 0(\varepsilon) \\ -a(1 - \cos(2N+1)\varepsilon/2) + b \sin(2N+1)\varepsilon/2 + 0(\varepsilon) \end{bmatrix}$$

and MLS-estimation of an amplitude vector is

$$E\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \frac{1}{2N \sin \varepsilon/2} \begin{bmatrix} a \sin(2N+1)\varepsilon/2 + b(1 - \cos(2N+1)\varepsilon/2) + 0(\varepsilon) + 0\left(\frac{1}{N}\right) \\ -a(1 - \cos(2N+1)\varepsilon/2) + b \sin(2N+1)\varepsilon/2 + 0(\varepsilon) + 0\left(\frac{1}{N}\right) \end{bmatrix}.$$

If in amplitude estimates we neglect the terms of order of magnitude $0(\varepsilon)$ and $0(1/N)$ then it is easy to get

$$\frac{\sqrt{\hat{a}^2 + \hat{b}^2}}{\sqrt{a^2 + b^2}} \approx \frac{|\sin N\varepsilon/2|}{N\varepsilon/2}.$$

Hence, even when one is treating with the signals without noise ($\sigma^2 = 0$) and the error in frequency evaluation is small the MLS-estimation of an amplitude vector converges to zero and the rate of its convergence to zero is defined by the function $|\sin N\varepsilon/2|/(N\varepsilon/2)$.

In the general case for arbitrary m, if we suppose that the errors in frequency evaluation $\varepsilon_k (1 \leqslant k \leqslant m)$ are essentially less than the difference between ω_i and ω_j $(i \neq j)$, then it is possible to show that the values of the corresponding elements $\hat{a}_k$, $\hat{b}_k$ are defined with the accuracy to $0(\|\varepsilon\|)$ only by values N and ε_k and are independent of the errors in the evaluation of other components of vector ω. In this case the ratio $\sqrt{\hat{a}_k^2 + \hat{b}_k^2}/\sqrt{a_k^2 + b_k^2}$ can be approximately defined as the function $|\sin N\varepsilon_k/2|/(N\varepsilon_k/2)$ for a sufficiently large value of N. Note, that this relation is independent of the values of the vector ω.

To compare the value of the ratio $\sqrt{\hat{a}_k^2 + \hat{b}_k^2}/\sqrt{a_k^2 + b_k^2}$ with the values of the function $|\sin N\varepsilon_k/2|/(N\varepsilon_k/2)$ for different values of ε_k and N we have performed a number of computational experiments. Here as an example we present the results of the simplest experiment.

We suppose that the signal model is

$$y_t = A \sin \omega t$$

and $\hat{A}$ is the amplitude A MLS-estimate when the frequency ω is evaluated with an error ε. In this experiment we examine the quantity $|\hat{A}/A|$ for error values $\varepsilon \in \{10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}, 10^{-12}\}$. The results of the experiment for the samples volume $N = 100, 200, \ldots 100 \times 2^{40}$ are shown in Figure 1. The plot of the ratio $|\hat{A}/A|$ changes for the fixed ε is presented by the continuous curves and the $|\sin N\varepsilon/2|/(N\varepsilon/2)$ values for corresponding $(N\varepsilon)$ are presented by dots. As follows by the figure,

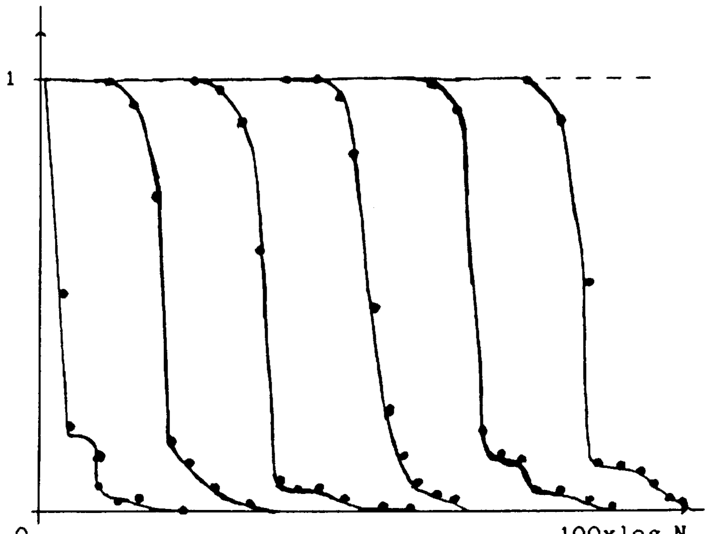

Figure 1 The comparison of the values $(|\sin N\varepsilon/2|)/(N\varepsilon/2)$ and $|\hat{A}/A|$.

the $|\hat{A}/A|$ and $|\sin N\varepsilon/2|/(N\varepsilon/2)$ values are similar for each of the considered values N and ε. Thus the experiment confirms our conclusions and we can use the analysis of the function $|\sin N\varepsilon/2|/(N\varepsilon/2)$ performance to investigate the ratio $|\hat{A}/A|$ performance. It is easy to see, that if ε is small and N value is not large, the product $(N\varepsilon)$ is in the neighborhood of zero and hence $|\sin N\varepsilon/2|/(N\varepsilon/2) \approx 1$. The experiment confirms the fact $\hat{A} \approx A$. When the values of N increases and $(N\varepsilon)$ becomes significant, $|\sin N\varepsilon/2|/(N\varepsilon/2)$ vanishes rapidly and as one can see the ratio $|\hat{A}/A|$ behaves in the same way.

The obtained results enable approximate evaluation of the domain of correct use of the existing GMDH harmonic algorithm. Let us assume the domain of correct application of MLS-estimation in the form of an inequality

$$E|\hat{A}/A| > 1 - \alpha.$$

As follows from the experiment, this domain can be defined approximately by the relation

$$\frac{|\sin N\varepsilon/2|}{N\varepsilon/2} > 1 - \alpha.$$

One can easily solve such inequality for each fixed value α.

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